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W.H.C. Bassetti
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FRACTAL WAVES in STOCK MARKET PRICES

A New Tool for Technical Analysis

Arthur von Waldburg,
A&M Trading Company, Inc.

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Abstract

An argument is made that the stock market bears similarities to non-linear dynamic processes which give rise to fractal structures. Stock price sequences are thought to be fractal in nature. A Fractal Wave Algorithm (FWA) for identifying the fractal structure of a serial stream of price data is presented in the context of a "classic" Elliott Wave formation. The FWA is used to determine the wave structure of recent Dow Jones Industrial Average (DJIA) prices based on hourly data from January 2, 1990. The FWA is shown to have value as a technical trading tool on hourly DJIA data from March, 1987 and on daily DJIA data from 1885.

Non-Linear Dynamic Processes and the Stock Market

It is common for securities and commodities analysts to assume that much of the observed price fluctuations in these markets are due to random processes acting on the price in the presence of longer term trends and changes in price equilibrium due to changes in fundamental supply and demand factors. This is a normal assumption to make because, after subtracting out underlying trends, the price movements do indeed appear random.

It is also worth considering that these price movements may be the result of nonlinear processes in the market place. There are a great many market participants, with complex sets of human relationships, motivations, and reactions. It would be astounding if all these human factors averaged out to a linear price mechanism.

If price movements near equilibrium were ruled by linear feedback mechanisms, price adjustments would be simply proportional to the amount the price were above or below equilibrium. The response of a linear system to small changes is usually smooth. The response to an external shock is generally a series of oscillations which decay in amplitude until equilibrium is again reached.

Nonlinear systems, however, often show transitions from smooth motion to chaotic, erratic, or apparently random behavior. The response of a nonlinear dynamic system to an external shock can take the form of persistent structures.

The possible behaviors of nonlinear systems may be extremely rich and complex. In particular, a series of numbers generated by a completely deterministic, but nonlinear, system could appear to be completely random when there is no noise in the system.

Professor Robert Savit of the University of Michigan pointed out in a recent article in The Journal of Futures Markets (Vol. 8, No. 3, 271-289) that a simple non-linear model of price movement can generate a price sequence that looks superficially random but is in fact chaotic, containing a good deal of hidden order. The information in the chaotic sequence may not be accessible to the researcher who uses traditional statistical methods which attempt to "smooth" noise out of the data. Other methods appropriate to nonlinear, chaotic systems, methods which take advantage of the "jumpiness" and "apparent disorder," may be more useful.

The advent of relatively cheap and powerful computers in the last ten to fifteen years has facilitated the study of previously intractable nonlinear systems. In fact the term, "experimental mathematics" has been coined to describe computer-based investigations of problems inaccessible to analytic methods. Many researchers making use of experimental mathematical techniques have discovered a relationship between nonlinear dynamic systems and the formation of "fractal" patterns. These patterns, which exhibit self-similarity at many scales, show up in the study of turbulent flow, the geology of oil recovery, dendritic growth in a solidification process, the development of mesoscale structures in metallurgy, the spread of disease, and the dynamic "rhythms" of the human heart.

<u>The Fractal Wave Algorithm</u>

One of the most widely followed methods of technical analysis of stock prices in the last decade, Elliott Wave Theory, makes reference to a "waves within waves within waves" structure of price movement. This concept of self-similarity at many scales in price structure is the same concept biologists and physicists are finding in the research mentioned above.

While I make no claim to being an expert at doing Elliott Wave analysis, my limited reading about the theory suggests that it is a "finite" theory, defining a limited number of price formations, which can be catalogued. This catalogue can be used to identify price patterns as they occur in the marketplace. Based on the current pattern being traced out at each scale, or "degree," future price movement can to some extent be predicted assuming the pattern currently being formed continues to completion.

My own attempts to anticipate the Elliott Wave analysis of one of the leading proponents of the theory have been unsuccessful, no doubt due in large part to my own lack of technical knowledge in the Elliott Wave science. Many followers of the Elliott Wave theory feel their own analysis is meaningful. There is often disagreement among the Elliott Wave analysts, however, leading me to believe it may require some art as well as science.

My experience is that the price patterns produced by the stock market (the research discussed here will be limited to the Dow Jones Industrial Average, DJIA) have an element of perpetual novelty which is inconsistent with a finite theory. I do believe, however, that the concept of self-similarity at many scales is extremely important and appropriate to the study of market price movements. I believe that price movements may be usefully studied as fractal structures arising in the context of nonlinear dynamics.

I have developed a method of identifying a fractal structure in stock price movement which borrows the concept of self-similar waves at many scales from Elliott Wave theory. This method, which I call the Fractal Wave Algorithm (FWA), starts with the lowest scale data available and "marks" the extreme high and low prices as wave points of higher scales on the basis of the number of layers of self-similarity between the wave points. An example of a well-known "regular" fractal curve will help describe the concept of the FWA.

The solid lines in Figure 1 comprise an equilateral triangle, the first step in development of a Koch Triadic Snowflake curve. The dashed lines illustrate the construction of the second step, wherein each side of the triangle is replaced by four line segments. Each of the new line segments is one-third the length of the side of the triangle and the four segments are arranged so that a new equilateral triangle projects from the middle of each of the old sides.

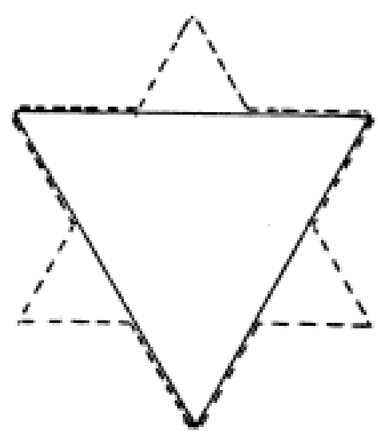

Figure 1. First Step in Koch Snowflake Development

If the process of replacing each side is repeated many times, the Koch snowflake curve is formed. Figure 2 shows the development carried a few more steps.

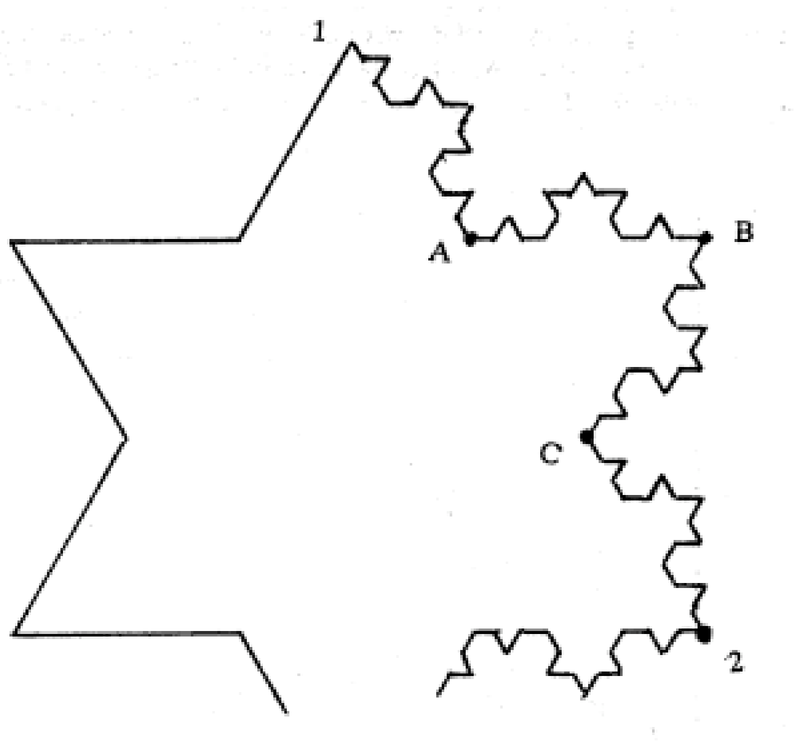

Figure 2. Koch Snowflake After Three Steps

The FWA is based on marking price curves such that a consecutive high and low pair marked as belonging at the same scale will have the same number of layers of self-similarity between them as any other consecutive pair marked at that scale. In Figure 2, the points labeled "A" and "B" could be said to have two layers of self-similar patterns between them. The pair of points labeled "B" and "C" also have two layers of self-similarity between them. The points labeled "1" and "2", however have three layers of self-similarity between them, the points "A", "B", and "C" make up that additional layer of self-similar pattern.

The FWA is a set of a few simple rules which allows any person marking a price curve to identify high and low wave points at many scales such that consecutive points identified at the same scale contain the same number of layers of self-similarity between them as any other consecutive points at the same scale.

We present here the rules of the FWA in the context of a "classic" Elliott Wave curve, to show that the FWA is a based on a general fractal wave approach within which the pattern analysis of Elliott Wave theory might be considered a more tightly specified area.

Figure 3 shows an idealization of an Elliott Wave curve constructed in a process like the one used to construct the Koch Snowflake. The solid lines represent a price advance followed by a price decline. The dashed lines show that the advance was made up of a "classic" five waves up and the decline was made up of three waves down. The waves are numbered "1-2-3-4-5-A-B-C." All of these wave points may be considered to be at the same scale. Points "0", "5", and "C" are significant at the next highest scale.

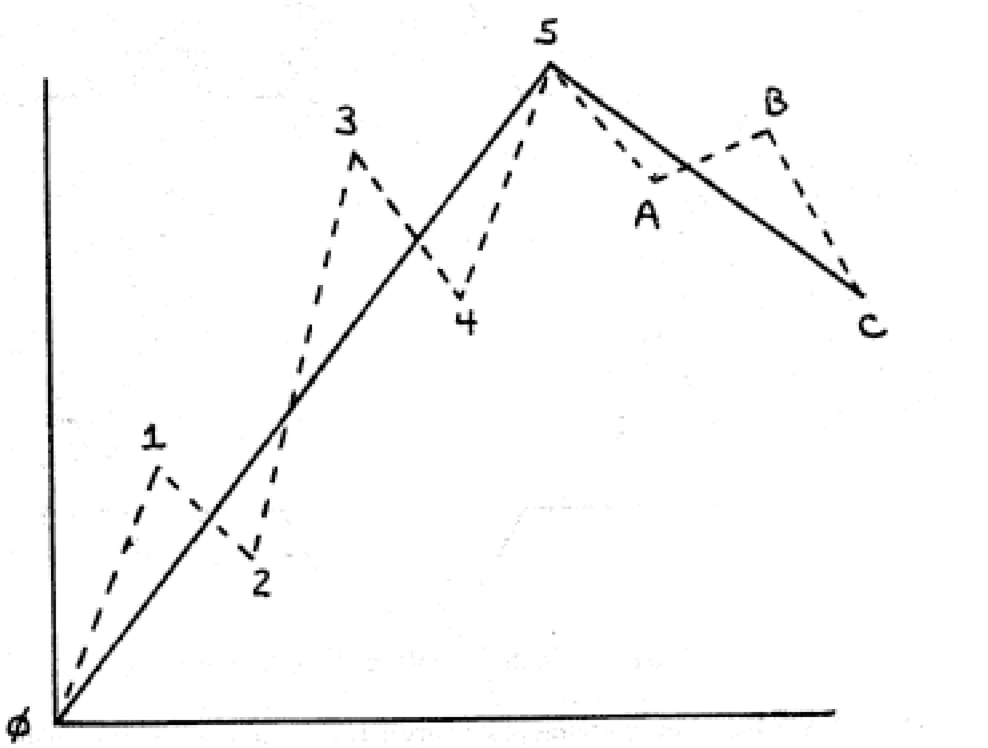

Figure 3. Classic Elliott Wave Development

If we continue the development of the Elliott Wave curve as we did the snowflake, we divide waves 1, 3, 5, A, and C into five waves each because they are in the direction of the wave at the next higher scale; and, we divide waves 2, 4, and B into three waves each because they are corrections to the wave at the next higher scale. The result is shown in Figure 4.

In developing the FWA, I focused on the fact that in the "classic" Elliott Wave, each advance was composed of a series of waves alternately making new highs and declining but not making new lows. Similarly, the corrective waves are made up of a series of waves alternately making new lows and rallying back, but not to new highs. Ignoring the number of waves required, you can see in Figure 3 that the wave up to point "5" is composed of "zig-zags" in the "up" direction and the wave from "5" down to "C" is composed of "zig-zags" in the "down" direction.

The FWA recognizes a "higher" scale wave by the fact that it is composed of "zig-zags" in one direction. When the first zig-zag in the opposite direction is completed, a new "higher" scale wave in the opposite direction has begun. The number of zig-zags at the lower scale in either direction is not relevant in the FWA.

In fact, ignoring the number of waves "required" in each direction is a simplification of Elliott Wave theory which allows the Fractal Wave concept to form the basis of a generalized wave theory; at the same time strictly determined so that any two analysts will mark a price curve the same, and less constrained so that any novel price formation that occurs is covered by the theory.

The second key element of the FWA is the recursive manner in which it is applied to a price series so that wave points at all degrees are identified as soon as lower scale layers of self-similarity become apparent. We can use Figure 4, below, to illustrate.

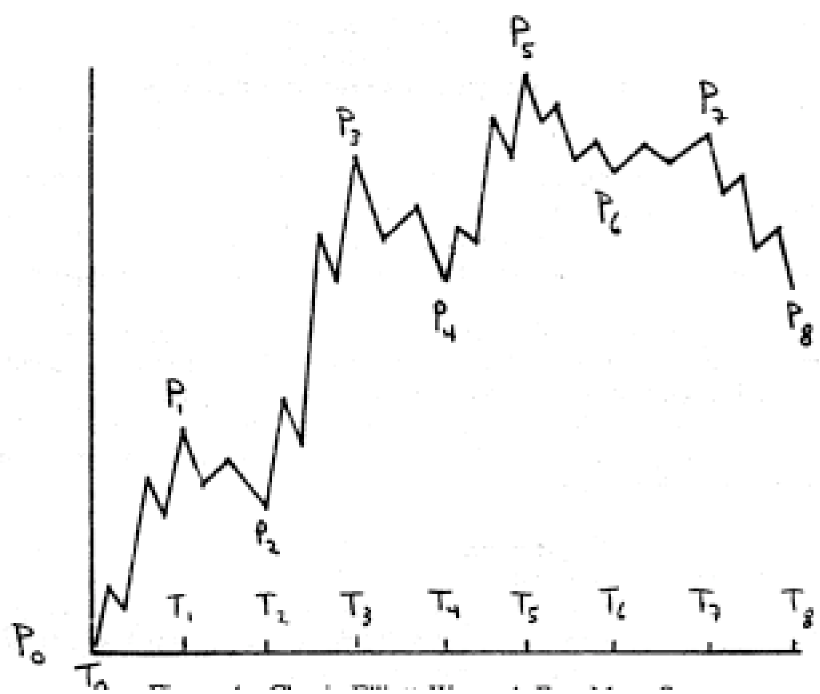

Figure 4. Classic Elliott Wave, A Few More Steps

Imagine the price curve in Figure 4 being traced out in real time. Let us call the data shown in the figure, "level-0" data. We will label successive highs and lows in the level-0 data as level-0 wave points. We will use the level-0 wave points to identify self-similar waves at "level-1". Level-1 wave points which complete zig-zags in one direction or the other will identify level-2 wave points. Etc., etc.

Starting from T_0 in Figure 4, the price advances upwards in zig-zag waves to P_1. The first zig-zag in the up direction is completed about half way to T_1. At the time when the first zig-zag of level-0 wave points is completed, we know the point at P_0 is a wave point at the next highest level, level-1.

At P_1 the price falls, then for the first time rises and falls again without making a new high. By time T_2, the price has completed a level-0 zig-zag in the down direction at price, P_2. When the zig-zag of level-0 wave points is completed in the down direction, we can mark the price, P_1, as a level-1 wave point. Now we have two level-1 wave points, P_0 and P_1.

A very important pattern occurs about half way between T_2 and T_3: As the price in Figure 4 completes a zig-zag up from point, P_2 we can mark P_2 as a level-1 wave point. At the <u>same</u> time, we have three level-1 wave points, P_0, P_1, and P_2, and a current price higher than P_1. A level-1 wave point at the current price or higher would complete a level-1 zig-zag in the up direction. We can anticipate with certainty that there will be a level-1 point at the current price or higher, so we can immediately mark the point at P_0 as a level-2 wave point.

Re-read the previous paragraph to make sure you understand how completion of a pattern at the lowest level stimulates the marking of wave points at higher levels. The completion of the level-0 zig-zag between T_2 and T_3 identifies the point at P_2 as a level-1 wavepoint, which contributes to the identification of P_0 as a level-2 wavepoint.

Obviously, the greater the significance (level) of a price high or low, the more <u>subsequent</u> price pattern is required for the significance to be recognized. The important thing about the FWA is that the lag required to recognize the importance of a high or low is dependent on layers of self-similarity in price pattern, not time. The FWA does not depend on scaler parameters such as 12 week cycles, five percent filters, 200 day moving averages or the like. It depends on <u>pure pattern</u>.

To continue with the example in Figure 4, note that the point at P_5 is recognized as a wavepoint at level-2 when the first zig-zag in the down direction is completed after the point at P_7. If the price were to advance in a similar fashion after P_8, and rise above the price at P_5, then P_8 would become a level-2 wavepoint and the required pattern elements to recognize P_0 as a level-3 wavepoint would be in place.

To summarize the FWA rules: Accept an available stream of price data as level-0 data. Mark alternating highs and lows in the price stream as level-0 wavepoints. When a level-0 zig-zag is completed in the up direction, mark the appropriate level-0 low as a level-1 wavepoint. Similarly, when level-0 zig-zags are completed in the down direction, mark the appropriate level-0 high as a level-

1 wavepoint. In the same way, use level-"k" zig-zags to identify level-"k+1" wavepoints.

Fractal Waves in the Dow Jones Industrial Average

The previous section presented the FWA in the context of a "classic" Elliott Wave price curve. Every technical analyst, Elliott Wave follower or not, is aware that the markets rarely trace out price patterns in that classic form.

Figure 5 is a chart of the Dow Jones Industrial Average (DJIA) based on a stream of prices recorded each hour from January 2, 1990 through January 26, 1990. Opening prices were not recorded; the first price recorded after the previous close was the 10:00 am (New York) price.

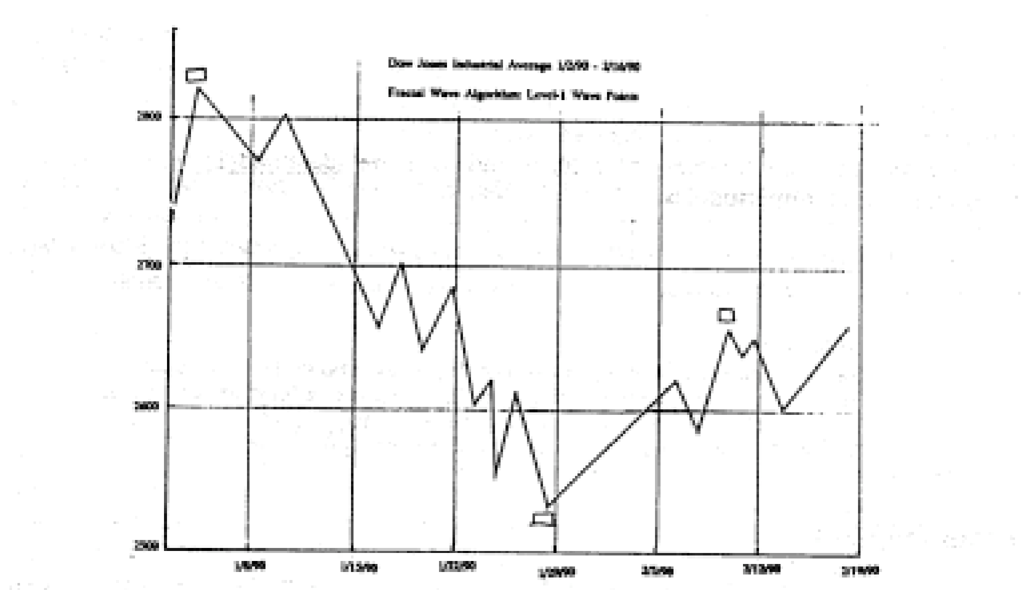

Figure 5. DJIA Level-1 Fractal Waves

Based on hourly data, there are 237 level-0 waves in the DJIA from January 2, 1990 till February 16, 1990. In Figure 5 only the level-1 wave points have been plotted. There are only 19 waves at level-1. As you look at Figure 5, remember that on the average, each wave shown is made up of twelve zig-zags running in the direction of the wave and no zig-zags in the opposite direction.

Since there is a series of level-1 zig-zags in the down direction from the January 3rd high at 2820.61, that point is a level-2 wavepoint, denoted by a square symbol at the point. There are two more level-2 wavepoints marked in Figure 5, the most recent on February 8 at 2655.63. Note that a penetration of level-2 low at 2533.11 on January 26 would raise the significance of the January 3rd high to level-3.

Value of Fractal Wave Structure as a Predictive Tool

The Stock Market may be driven by non-linear dynamic processes which give rise to fractal price structures. The FWA is a generalized technique for labeling price highs and lows as to their significance within a fractal price structure. The practical question is, "Does this discovered fractal structure provide information in real time which can be used to make money in the stock market?"

In an attempt to shed some light on that question, I have conducted a series of research studies using hourly DJIA data from April, 1987 to February, 1990, and daily DJIA data from 1885 to February, 1990.

My conjecture is that a wavepoint identified by the FWA as a level "K" high is a significant indication that prices will move lower; that a wavepoint identified by the FWA as a level "K" low is a significant indication that prices will move higher. Of course, by the design of the algorithm, the highs are higher than the lows; as with all trading tools, the success hinges on whether these highs and lows are identified in time to provide successful trading opportunities.

To test this conjecture, the DJIA data was run through the FWA. Each wave level was tested in the same fashion, simultaneously, according to the following rules:

At level "K":
Take the most recent wavepoint at level "K+1" to indicate the price trend.
Take the most recent wavepoint at level "K" to indicate a "buy" or "sell" signal.
Be "long" if the "K+1" trend is up and the "K" signal is "buy."
Be "short" if the "K+1" trend is down and the "K" signal is "sell."

Hourly DJIA Data

The results for hourly data from March, 1987 are given in Table 1. You should note that there was a +352 point bias in the hourly data. That is, the first price was 352 points

Wave Level	Number of "Trades"	Score (DJIA Points)
0	1402	1659
1	266	1040
2	62	3

3	11	561
4	1	593

Table 1. FWA Predictive Score: DJIA Hourly from March, 1987

lower than the final price. The lowest price was 1747; the highest price was 2814. There were no identified wavepoints at level 5 or higher. Only one wavepoint was identified as level 4, the low at 1747.

There were 11 trades identified at level 3. The total score for being long or short with the level 3 direction, when it was in the direction of the level 4 wave was 561 Dow points. The average trade at level 3 resulted in a 51 point profit. Five of the level 3 trades were profitable. The average gain was 233 points; the average loss was 101 points. This would appear to be potentially useful information, even after allowing for "trading costs" such as execution slippage and commissions. The addition of standard stop loss techniques could possibly yield an attractive trading method.

The results for levels lower than level 3 are similar at each level. In each case there are too many "trades" identified with, on average, too little price movement to provide attractive opportunities. The average gain or loss per trade is one to two Dow points. I would conclude on the basis of this small amount of data that there is insufficient information produced by the FWA at levels 0, 1, and 2 on hourly data to provide profitable trading opportunities.

Daily Data

The results for daily data from 1885 are given in Table 2. We see a similar pattern to that obtained from the hourly data: The number of trades decreases as would be expected with increasing wave level, and the "score" decreases to a minimum, then increases with increasing wave level. For the results shown here significant trading opportunities from daily data emerge at level 4, where the average score per trade is almost 200 Dow points.

Wave Level	Number of Trades	Score (DJIA Points)
0	7734	5161
1	1277	624
2	232	105
3	44	1059
4	10	1986
5	1	2526

Table 1. FWA Predictive Value: DJIA Daily from 1885

Summary

In summary, I would conclude that it is possible to recognize profitable trading opportunities by identifying a fractal wave structure in stock market price data. The FWA is a useful addition to the arsenal of tools used by technical analysts.

There are many areas in which my research is continuing, including analysis of "tick" data for a wide range of commodity markets, development of more sophisticated trading "systems", and real-time trading of some preliminary systems.